RELATIVE MOTION IN TWO DIMENSIONS

<< methods of solving problems on RM in 2D>>

METHOD I : APPLYING LAW OF INDEPENDENCE OF DIRECTIONS

EXAMPLE :

A boy is moving with uniform speed 30 km/hr towards east and a girl is moving with uniform speed 4 km/hr towards north, while having morning walk in a garden. The boy is continuously looking at the girl. What is the velocity of girl as seen by boy ?

Sol. $U_{xgirl} = 0$, $U_{ygirl} = 4$ km/hr, $U_{xboy} = 3$ km/hr, $U_{y\,boy} = 0$

$U_{xgirl.boy} = U_{xgirl} - U_{xboy} = 0 - 3 = -3$ km/hr.

$U_{ygirl,boy} = U_{ygirl} - U_{yboy} = 4 - 0 = 4$ km/hr.

so $U_{girl,boy} = U_{xgirl,\,boy}\,\hat{i} + U_{ygirl,boy}\,\hat{j} = -3\hat{i} + 4\hat{j}$

$\therefore$ $|Urel| = \sqrt{3^2 + 4^2} = 5$ km/hr, $\tan\theta = -\frac{4}{3}$ (roughly towards north-west)

METHOD II : BY DRAWING THE VECTOR DIAGRAM AND ANALYSING IT WITH ELEMENTARY KNOWLEDGE OF TRIGONOMETRY/GEOMETRY.

ILLUSTRATIVE CASES

THE MAN AND THE RAIN PROBLEM

The aim is to determine the angle at which the man should hold the umbrella to prevent himself from wetting.

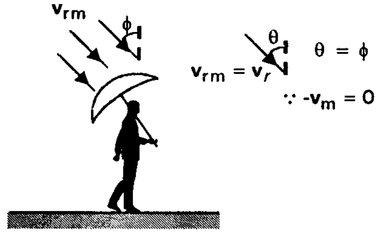
Here, v_r = velocity of rain w.r.t. ground

v_m = velocity of man w.r.t. ground

v_{rm} = velocity of rain w.r.t. man

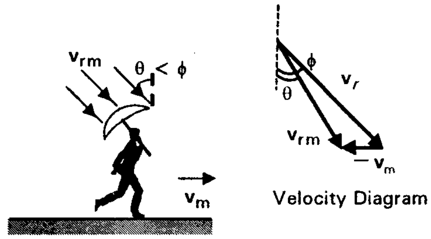
The answer to the problem is that he should hold the umbrella in the direction from where the rain appears to be falling.

1. The man is stationary and the rain is falling at his back at an angle ϕ with the vertical.



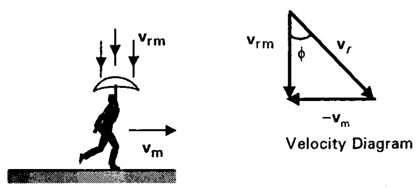
2. The man starts moving forward. The relative velocity of rain w.r.t. man shifts towards vertical direction (not away from it as a general misconception may be).

$|v_m| < |v_r \sin \phi|$ = horizontal component of rain velocity.



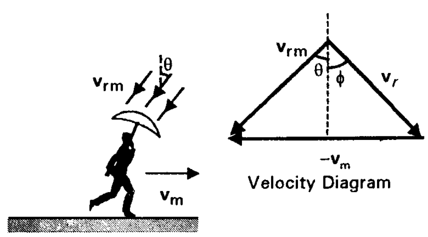
3. As the man further increases his speed, then at a particular value, the rain appears to be falling vertically.

$|v_m| = |v_r \sin \phi|$



4. If the man increases his speed further more, then the rain appears to be falling from the forward direction.

$|v_m| > |v_r \sin \phi|$



RIVER AND BOAT PROBLEM

The various parameters are :

v_r = velocity of river currents w.r.t ground

v_s = velocity of the swimmer w.r.t. river

V_s = resultant velocity of swimmer with respect to ground.

b = width of the river.

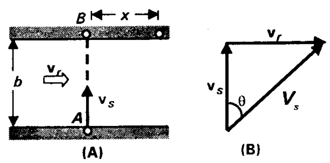
The boat crosses the river due to its component of velocity perpendicular to the river currents.

RELATIVE MOTION IN TWO DIMENSIONS

(I) CONDITION, WHEN THE BOAT CROSSES THE RIVER IN SHORTEST PERIOD OF TIME

The boat is moving normal to the river currents. The time taken to cross the river is minimum and is given by.

$$t_{min} = \frac{b}{v_s}$$



The boat does not reach exactly at the opposite point B on the bank but reaches at the point C, which is at a distance x from the point B.

$$x = v_r\, t_{min} = \left(\frac{v_r}{v_s}\right) b$$

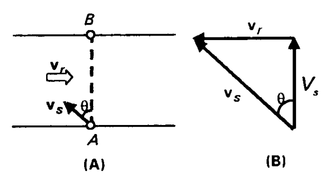
(II) CONDITION, WHEN BOAT WANTS TO REACH POINT B I.E. AT A POINT JUST OPPOSITE FROM WHERE IT STARTED

The boat moves at an angle θ with the normal against the river currents and it is able reach the exactly opposite point B on the bank. The angle θ is given by.

$$\tan\theta = \frac{v_r}{V_s} = \frac{v_r}{\sqrt{v_s^2 - v_r^2}}$$

The time taken to cross the river is :

$$t = \frac{b}{v_s\cos\theta} = \frac{b}{V_s} \quad \text{or} \quad t = \frac{b}{\sqrt{v_s^2 - v_r^2}}$$



VELOCITY OF APPROACH.

velocity of approach ($v_{app.}$) of particle A w.r.t. B is the component of the relative velocity of A w.r.t. B along the line joining A and B and is directed towards B.

1. It's rate of decrease of separation if A is moving towards B.
2. If velocity of approach is constant then

$$\text{time of collision} = \frac{\text{initial separation}}{\text{velocity of approach}}.$$

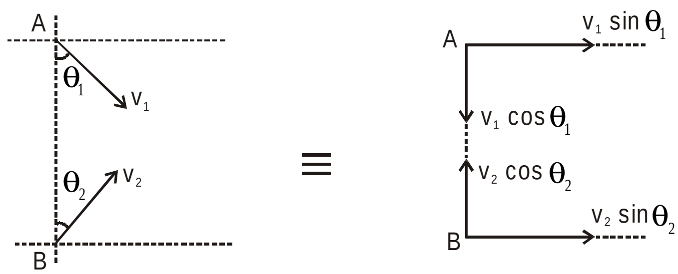
RELATIVE MOTION IN TWO DIMENSIONS

3. If v_{app} is variable and t_o = time taken for collision then

initial separation $s_o = \int_o^{to} v_{app.}\, dt$

ILLUSTRATIVE CASES

(1)

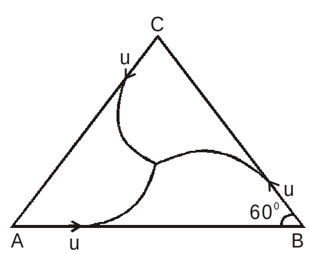


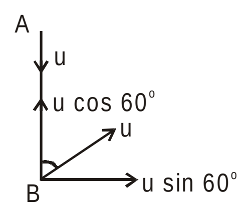
$(v_{app} = v_1 \cos \theta_1 + v_2 \cos \theta_2)$

EXAMPLES BASED ON RELATIVE MOTION 2-D

Ex.1 If three particles A, B, C are situated at vertices of an equilateral triangle of side d. They start moving all together at the same instant with the same speed u such that A continuously moves towards B, B continuously moves toward C, C continuously moves towards A, then find the time of collision.

Sol. $v_{app} = u + u \cos 60 = \frac{3u}{2}$

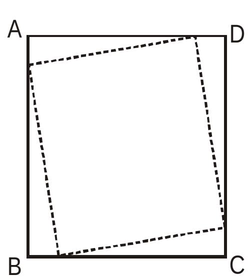


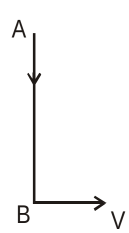


$\therefore$ time of collision = $\frac{\text{initial separation}}{v_{app}} = \frac{2d}{3u}$

Ex.2 If triangle is replaced by square of side a then what is the time of collision ?

Sol.





$v_{app} = v + 0 = v$

$\therefore$ time of collision = $\frac{a}{v}$.

RELATIVE MOTION IN TWO DIMENSIONS

LEVEL # 1

Q.1 A boat is moving with velocity of $3\hat{i} + 4\hat{j}$ in river and water is moving with a velocity of $-3\hat{i} - 4\hat{j}$ with respect to ground. Relative velocity of boat with respect to water is-

(A) $-6\hat{i} - 8\hat{j}$ (B) $6\hat{i} + 8\hat{j}$ (C) $8\hat{i}$ (D) $6\hat{i}$

Q.2 For a body moving with relativistic speed, if the velocity is doubled, then-

(A) Its linear momentum is doubled
(B) Its linear momentum will be less than double
(C) Its linear momentum will be more than double
(D) Its linear momentum remains unchanged

Q.3 A river is flowing from W to E with a speed of 5 m/min. A man can swim in still water with a velocity 10 m/min. In which direction should the man swim so as to take the shortest possible path to go to the south ?

(A) 30º with downstream (B) 60º with downstream
(C) 120º with downstream (D) South

Q.4 A train is moving towards east and a car along north, both with same speed. The observed direction of car to the passenger in the train is-

(A) East-north direction (B) West north direction
(C) South east direction (D) None of these

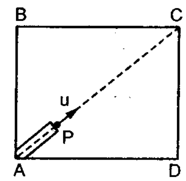
Q.5 An express train is moving with a velocity v_1. Its driver finds that another train is moving on the same track in the same direction with velocity v_2. To escape collision, driver applies a retardation a on the train. The minimum time of escaping collision will be-

(A) $t = \dfrac{v_1 - v_2}{a}$ (B) $t_1 = \dfrac{v_1^2 - v_2^2}{2}$ (C) None (D) Both

LEVEL # 2

MORE THAN ONE CHOICE MAY BE CORRECT :

Q.1 A large rectangular box falls vertically with an acceleration a. A toy gun fixed at A and aimed towards C fires a particle P.



(A) P will hit C if a = g.
(B) P will hit the roof BC if a > g.
(C) P will hit the wall CD or the floor AD if a < g.
(D) May be either (A), (B) or (C) depending on the speed of projection of P

Q.2 To a man running at a speed of 20 km/hr, the rain drops appear to be falling at an angle of 30º from the vertical. If the rain drops are actually falling downwards, their velocity in km/hr is.

(A) $10\sqrt{3}$ (B) 10 (C) $20\sqrt{3}$ (D) 40

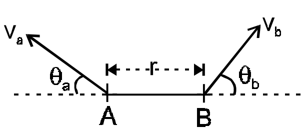
Q.3 D is the width of a river, v_R is the velocity of river and v_S is the velocity of swimmer. The shortest route to cross the river is.

(A) D when $v_S > v_R$
(B) $\frac{Dv_R}{v_S}$ when $v_R > v_S$
(C) At right angle to river when $v_S > v_R$
(D) At an angle $a = \tan^{-1}\frac{v_S}{\sqrt{v_R^2 - v_S^2}}$ when $v_R > v_S$

Q.4 A car with a vertical wind shield moves along in a rain storm at speed of 40 km/hour. The rain drops fall vertically with a terminal speed of 20 m/sec. The angle at which the rain drops strike the wind shield is

(A) $\tan^{-1}(5/9)$ (B) $\tan^{-1}(9/5)$ (C) $\tan^{-1}(3/2)$ (D) $\tan^{-1}(2/3)$

Q.5 Two particles A and B are moving as shown in the figure. At this moment of time the angular speed of A with respect to B is



(A) $(v_a + v_b)/r$
(B) $(v_a - v_b)/r$
(C) $(v_b \sin\theta_b - v_a \sin\theta_a)/r$ in anticlockwise direction
(D) $(v_b \sin\theta_b + v_a \sin\theta_a)/r$ in anticlockwise direction

ANSWER KEY

LEVEL # 1

Que.	1	2	3	4	5
Ans.	B	C	C	B	A

LEVEL # 2

Que.	1	2	3	4	5
Ans.	A,B,C	C	A,B	A	C